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Nonlinear Elastic Wave Interaction in a Sandstone Bar: A Summary of Recent Pulse-Mode Experiments

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SUMMARY

We have performed nonlinear pulse propagation experiments in a 3.8 cm diameter rod of Berea sandstone 1.8 m long at ambient conditions. Unlike earlier studies, we measured acceleration and not displacement. Moreover, we detected 2nd *and* 3rd harmonic growth at smaller strain amplitudes than were observed previously (10^{-7}). Harmonic growth at identical strain amplitudes has also been noted in resonance studies using the same rock type. Current measurements are underway with the rod in vacuum where the wave attenuation is less and the conditions can be carefully controlled. Ultimately, we wish to test the validity of current analytic and numerical models for nonlinear propagation in microcracked materials.

INTRODUCTION

In previous laboratory experiments Meegan et al.[1] demonstrated that harmonics of pure tone signals are nonlinearly generated along the wave propagation path in a sandstone bar at strain levels as low as 3 x 10^{-6}. The experiments seemed to confirm predictions from perturbation theory[2] that the second harmonic amplitude grows linearly with propagation distance, with the square of the input frequency, and with the square of the fundamental amplitude. The measurements were made in a 2 m long x 6 cm diameter rod of Berea sandstone at ambient conditions, with a piezoelectric source at one end and displacement detectors embedded in the rod within small boreholes drilled at various points along the rod axis. Model studies have recently been conducted from a solution to the 1-D nonlinear equation of motion using an iterative Green function method where a perturbative solution was found to second order in the nonlinearity.[3,4] The solution includes visco-elastic, linear attenuation. The model study is in agreement with the experimental observations of Meegan et al. [see Van Den Abeele et al., these Proceedings]. Recent resonance experiments[5] with similar samples, however, suggest a somewhat different model of the nonlinear elasticity inherent in rock samples may be more appropriate.[6] Hysteresis and end point memory, for example, may prove to be very important. Hence, we expand on the earlier experimental work and add to the observations already published in an effort to determine the limits of the current analytical models of nonlinear wave propagation in rock.

EXPERIMENTS

The sample used in these latest experiments is a nearly homogeneous but anisotropic, 1.8 m long, 3.8 cm diameter rod of Berea sandstone (Cleveland Quarries). Both ends were machined flat, perpendicular to the axis along the rod. A 3.8 cm diameter PZT-4A piezoelectric disk and a Tantalum inertial backload were epoxied onto one end to form the source. The other end was left free. In contrast to earlier experiments where the detectors were placed inside the rock, we chose to mount several high frequency B&K 8309 accelerometers (using a cyanoacrylate glue and an activator) directly to the outside surface of the rock, each oriented along the axial direction. In the previous experiments, wave scattering from the detectors was a critical problem. Accelerometers leave the rock unaltered. Moreover, at the higher harmonic frequencies typically seen in these nonlinear experiments, accelerometers are more sensitive than displacement sensors by a factor of ω^2.

The electronics attached to the source and receivers are as follows. An Analogic 2020 arbitrary function generator is the signal source. It is programmed to repeatedly output a pulse with a variable gaussian-shaped envelope. The output of the 2020 is fed into a Haffler Pro5000 audio amplifier which is connected to the piezoelectric disk via a transformer. Measurements of electronic harmonic distortion at the piezoelectric for all the experiments discussed here show that the harmonics were all more than 60 dB below the fundamental. Each accelerometer is fed into a B&K 2635 Charge Amp and then to a LeCroy 9420 Digitizing Oscilloscope. Signal-to-Noise ratios were improved by periodically pulsing the source and using standard linear averaging techniques.

The choice of operating frequencies was limited by the length of the bar and the possibility of exciting unwanted higher order modes. The propagation speed of the lowest longitudinal mode (or Young's mode) we wished to excite is about 2000 m/s for this sample. To obtain enough cycles for analysis before the arrival of the reflected pulse requires source frequencies above about 10 kHz. Accelerometer bandwidth limits and cutoff frequencies of the higher order longitudinal and torsional modes, on the other hand, place an upper limit on source frequencies. Cutoff frequencies for these modes were calculated from the rod geometry and bar and shear wave speeds. We found the next higher longitudinal and torsional mode are permitted (by rod geometry) at frequencies greater than about 35 kHz and 55 kHz, respectively. One more item limits the highest source frequency: the accelerometers have a flat (magnitude and phase) response to about 55 kHz, assuming each is mounted perfectly. For these reasons, source frequencies were kept between 10 and 20 kHz.

LINEAR MEASUREMENTS

During the initial linear measurements, we (re)discovered something known to Rayleigh, "The difficulty of exciting purely longitudinal vibrations in a bar is similar to that of getting a string to vibrate in one plane."[7] In fact, at our source frequencies, the lowest order longitudinal and torsional modes are *both* permitted along with a host of flexural modes. Although flexural modes are possible, they typically propagate with very slow speeds, are dispersive, and thus are easily distinguished from the longitudinal and torsional modes. While it is true that our source condition does not favor torsional mode excitation, we nevertheless found that certain source frequencies do, in fact, readily excite a strong mode that propagates at the torsional (shear) velocity (see, for example, Kwun and Teller[8] for another instance); naturally we avoided those source frequencies. Care was also taken in the orientation of the accelerometers on the rock. Each accelerometer has an axis of maximum transverse sensitivity. In an effort to obtain the best measure of torsional mode response, we oriented the accelerometer's transverse axis to maximize the signal from motion of the torsional mode.

The linear experiments also revealed that the lowest longitudinal mode does not develop immediately after it is emitted by the source. Typically, we did not observe the characteristics of Young's mode until the wave had propagated a distance of about one to two wavelengths. Thus, for the experiment described here, all measurements were made at a distance greater than 20 cm from the source. The solution of Van Den Abeele et al. [these Proceedings] can easily be marched out from this distance using the measured signal at 20 cm as a source. Work is being undertaken along these lines.

RESULTS

Several experiments were performed at very low strain amplitudes, typically around 10^{-8}. Although a second harmonic was present and grew with source amplitude, rock inhomogeneities and lack of sufficient signal-to-noise ratio in our digitizer did not allow us to separate nonlinear second harmonic growth from source distortion. At larger strain amplitudes, however, we did observe nonlinear distortion and harmonic growth.

Figure 1 shows two time series and their corresponding spectra. The lower plots are time and frequency domain representations of a low amplitude 15 kHz pulse after it has travelled 60 cm in the sandstone. The topmost plots illustrate a large amplitude, clearly *nonlinear* 15 kHz pulse at the same propagation distance. Distortion is apparent in both the waveform and spectrum. Prominent second, third, fifth and seventh harmonics can be seen. The corresponding peak strain amplitudes for this high and low amplitude sets of data are 2×10^{-6} and 7×10^{-9}, respectively.

We also performed an experiment similar to one described by Meegan, et al.[1] and measured growth of the second harmonic as a function of distance; moreover, we were even able to measure the growth of the third harmonic. Because of source contributions, wave attenuation, accelerometer mounting irregularities, and local inhomogeneities in the rock (commonly called *site response*), an absolute measure of harmonic growth as a function of distance is difficult. However, it *is* possible to correct for all of the above problems for each harmonic by measuring the response of each accelerometer to a small, linear signal propagating at those frequencies along the same distances. The ratio of the nonlinear amplitude to the small signal amplitude—the spectral ratio—effectively cancels out source and site response and the effect of attenuation (see the discussion in Meegan et al. for details). At small wave amplitudes, a plot of such a ratio as a function of distance yields a flat, straight line. At larger wave amplitudes, theory predicts that the harmonics should grow linearly

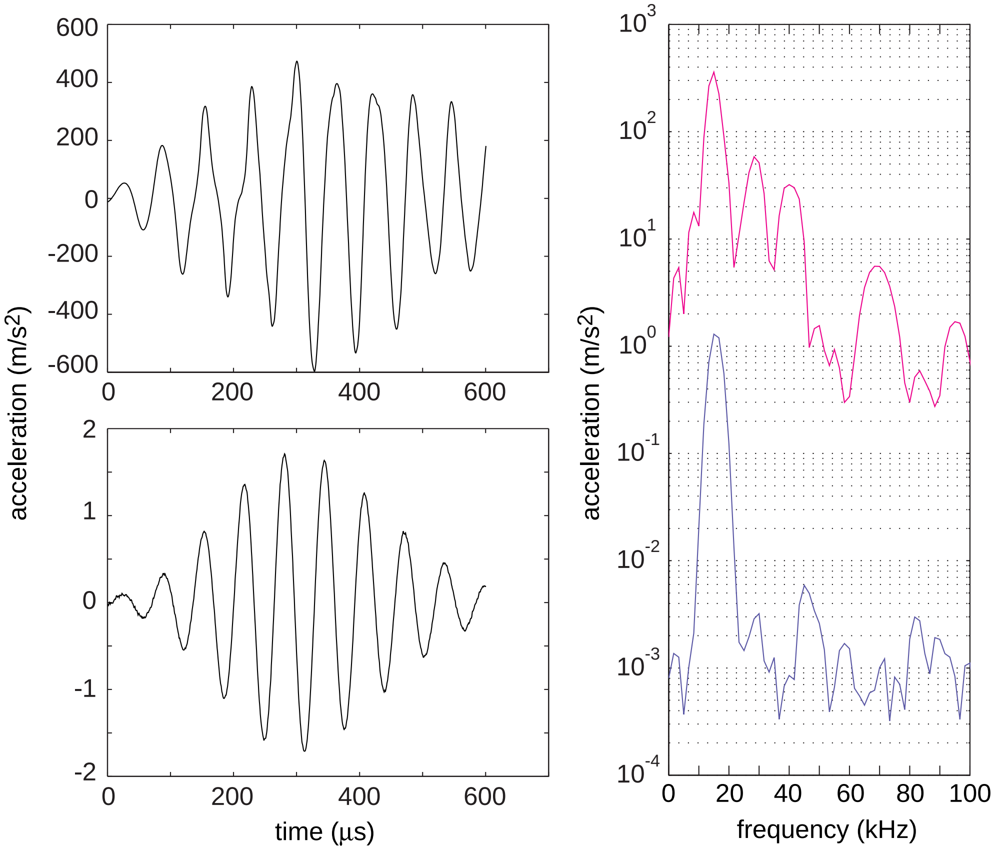

Figure 1: Time and frequency response in sandstone at large (top) and small (bottom) drive levels.

with distance. The results are shown in Figure 2 for a source frequency of 12.4 kHz and peak strain amplitude of 2 x 10^{-7}. Spectral ratios (denoted R2 and R3) of both the second and third harmonics were taken at each distance and plotted. Squares indicate the growth of the second harmonic, triangles indicate the growth of the third harmonic. Unfortunately, the errors in the method and sparseness of data do not allow us to deduce the functional form of our measured third harmonic growth. The error bars are significantly smaller than the spread in each case, however. We are currently planning additional measurements to specifically examine this growth.

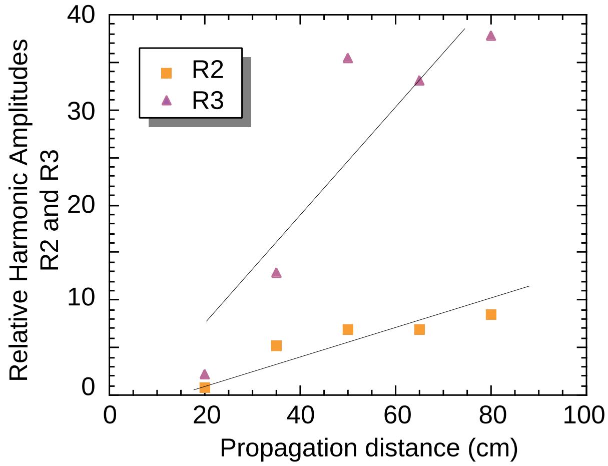

Figure 2: Harmonic growth as a function of distance

As indicated throughout the paper, the work is currently underway. Ongoing measurements should (1) allow us to determine how the third harmonic grows with distance, (2) give us a better measure of the nonlinear coefficients in the stress-strain equation of state, and (3) permit us to compare the perturbation and discontinous theories. We have also built a vacuum chamber for the rod. Measurements in vacuum produce more repeatable results, and less water vapor means much less wave attenuation and larger harmonic amplitudes. Our initial work with the rod in vacuum shows promise as well.

CONCLUSIONS

Several experiments have been performed and are currently underway studying nonlinear pulse propagation in a sandstone bar. Results show that the effects of nonlinearity are apparent at even lower strain amplitudes than were measured in earlier experiments. Additionally, we find both the second *and* third harmonic grow with distance. Continuing work should allow us to fully explore the validity of current mathematical and numerical models of such a highly nonlinear material.

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